Sustainable Transmission Planning

Afzal S. Siddiqui[a] Makoto Tanaka[b] Yihsu Chen[c]

[a] Stockholm University and Aalto University, e-mail address: asiddiq@dsv.su.se

[b] National Graduate Institute for Policy Studies

[c] University of California at Santa Cruz

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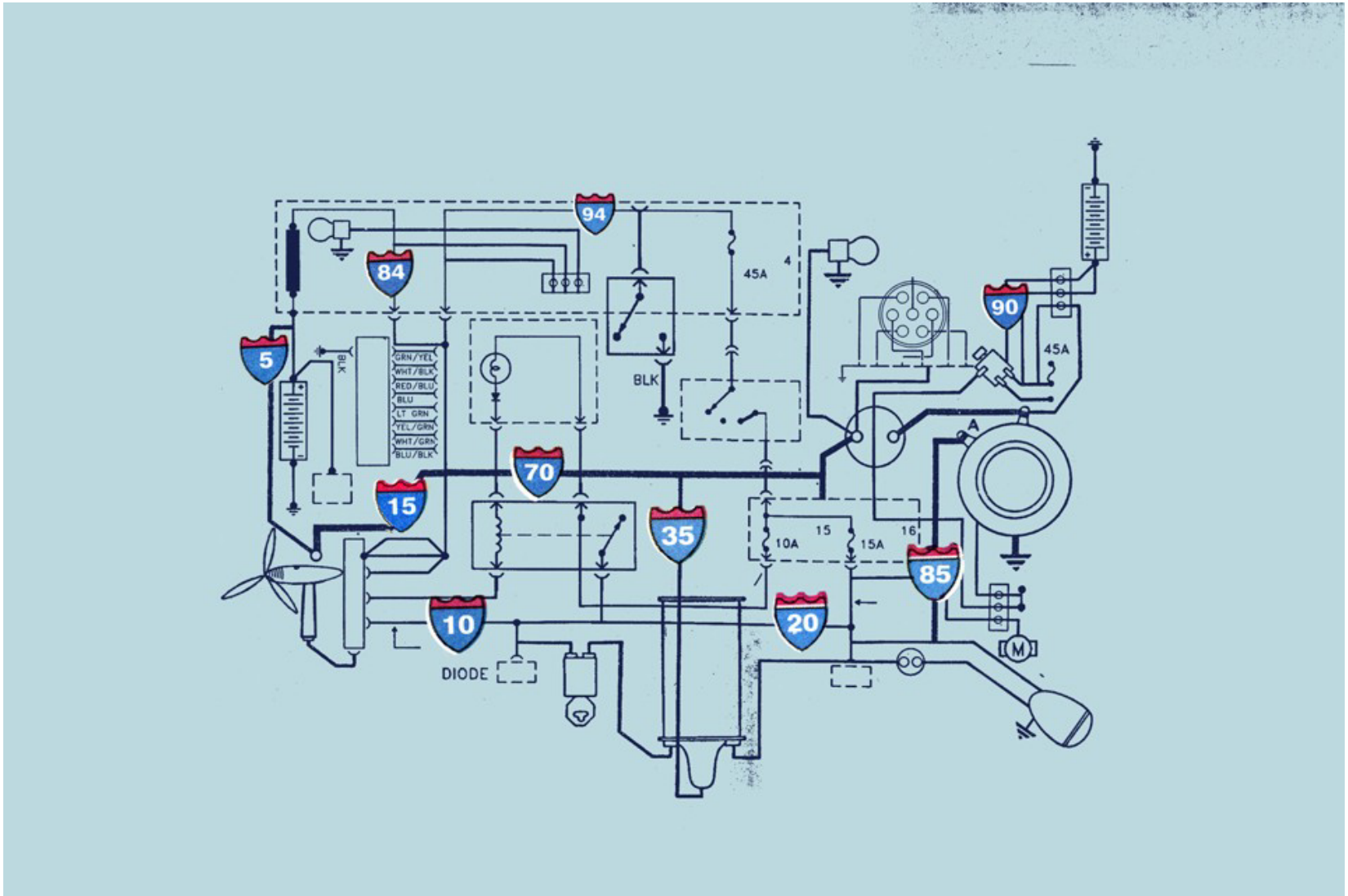
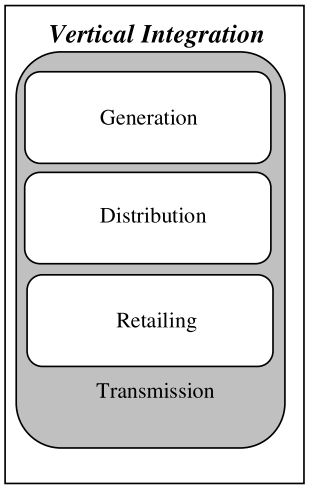
Introduction

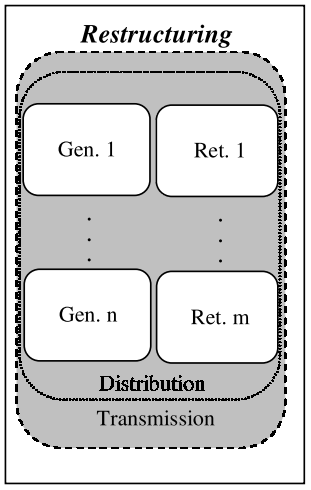
"The Energy Interstate"

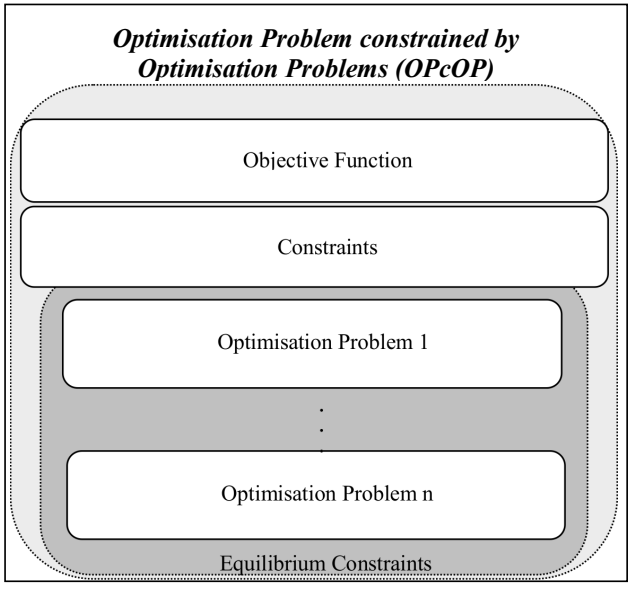
The Atlantic, June 2016

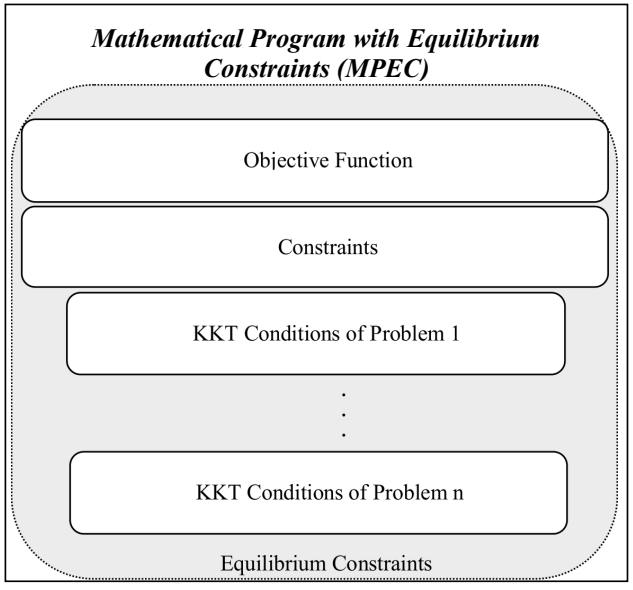


Evolving Paradigms and Models









Deregulation and Decarbonisation

- Regulated paradigm
  - Single decision maker
  - Single-level optimisation model for planning
- Post-restructuring
  - Imperfect competition
  - Endogenous price formation
  - Greater focus on variable renewable energy sources (VRES)
- Policymaker's dilemma
  - Comply with international treaties, e.g., European Green Deal
  - Yet, cannot be seen to interfere with industry
  - Delicate balance between providing incentives to guide a sustainable energy transition and blunting the market
  - Need to understand the implications of strategic behaviour when designing markets or setting carbon taxes
- Analysis of Environmental Policy in the Power Sector: Equilibrium Methods and Bi-Level Modeling

Transmission Planning in Deregulated Electricity Industries

- Optimisation in regulated paradigm (Garver, 1970) and real-world case studies (Hobbs, 1995)
- Borenstein et al. (2000) demonstrate the potential for enhanced competition from transmission expansion in a two-node Cournot duopoly model
- Sauma and Oren (2009) similarly examine the impact of financial transmission rights (FTRs)
- Sauma and Oren (2006) have tri-level models that illustrate the complexity for the transmission system operator (TSO) to obtain politically feasible transmission expansion
- Bi-level transmission and VRES investment: Baringo and Conejo (2012) and Maurovich-Horvat et al. (2015)
- Hobbs (2012) explores policy dilemmas with RE integration
- Environmental economics literature examines the efficiency of policies under market power (Barnett, 1980; Requate, 2005)

An Analytical Approach

- **Siddiqui et al. (2019) use a stylised two-node model to understand economic and environmental tradeoffs analytically**
- **Take the perspective of a welfare-maximising TSO that internalises the cost of damage from emissions**
- **In Norway,** *koordinert og trinnvis utvikling Statnett skal forsikrer seg om at det vil bli realisert en tilstrekkelig mengde vindkraft*
- **Allow for strategic behaviour (Cournot oligopoly) or not (perfect competition) by lower-level producers**
- **Compare different market settings: central planner (CP), perfect competition (PC), and Cournot oligopoly (CO)**
- **Sustainable transmission investment is curtailed under PC**
- **A full carbon tax imposed on industry under PC results in a first-best solution**
- **However, a carbon tax under CO may actually worsen welfare** *vis-à-vis* **doing nothing**

Detailed Model of Transmission Planning in a Deregulated Industry

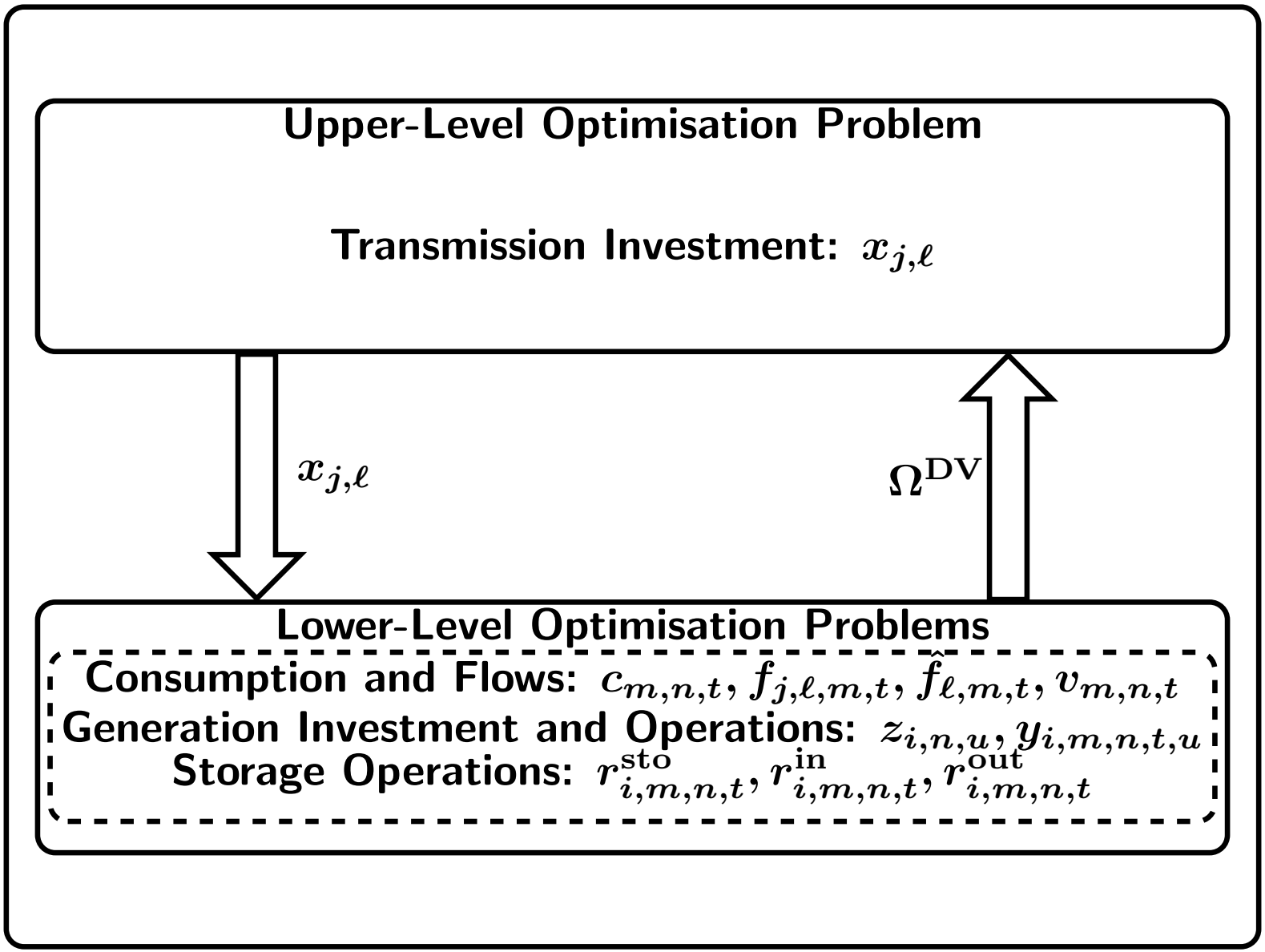
- Expand the Siddiqui et al. (2019) framework by introducing realistic features of power systems
  - Kirchhoff's laws approximated via linearised DC load flows
  - Heterogenous firms with portfolios of power plants
  - Intermittent VRES output
- Re-formulate the bi-level problem as a single-level mathematical program with primal and dual constraints (MPPDC) and subsequently as a mixed-integer quadratically constrained quadratic program (MIQCQP)
- Investigate the effectiveness of a carbon tax in aligning private and social incentives in a realistic test network
  - How is transmission planning affected by the damage cost?
  - Could a full carbon tax be actually welfare diminishing under CO?
  - What is the impact of greenfield versus brownfield assumptions?

Mathematical Formulation

Assumptions

- Representative weeks $m \in \mathcal{M}$ with weights W_m comprising time periods $t \in \mathcal{T}$
  - $n \in \mathcal{N}$ nodes with consumption $c_{m,n,t}$ (MWh) and inverse demand $A_{m,n,t} - Z_{m,n,t}c_{m,n,t}$ (€/MWh)
  - $\ell \in \mathcal{L}$ transmission lines with initial capacity $K^{\text{trn}}_{j,\ell}$ (MW) and susceptance $B_{j,\ell}$ (S) in capacity level $j \in \mathcal{J}_\ell$
- Welfare-maximising TSO internalises damage cost ($D \geq 0$)
  - Binary choice for each line and level $x_{j,\ell} \in \{0, 1\}$ with capacity size $K^{\text{trn}}_{j,\ell}$ (MW) and amortised cost $C^{\text{trn}}_{j,\ell}$ (€)
- Firms' open-loop profit maximisation (Wogrin et al., 2013)
  - Investment $z_{i,n,u}$ (MW) at amortised cost $C^{\text{gen}}_{i,n,u}$ (€/MW)
  - Operations $y_{i,m,n,t,u}$ (MWh) at cost C^{opr}_u (€/MWh)
  - Unit $u \in \mathcal{U}_{i,n}$ has availability factors, $\underline{G}_{m,n,t,u}$ and $\overline{G}_{m,n,t,u}$, ramp rates, R^{up}_u and R^{down}_u, and emission rate F_u (t/MWh)
  - Storage operations via $r^{\text{sto}}_{i,m,n,t}$, $r^{\text{in}}_{i,m,n,t}$, and $r^{\text{out}}_{i,m,n,t}$
  - Welfare-maximising ISO clears market via voltage angle $v_{m,n,t}$ (rad) to control realised power flow $\hat{f}_{\ell,m,t}$ (MW)

Bi-Level Framework



MPPDC Re-Formulation

MIQCQP resolution of the bi-level problem:

$$\underset{\{x_{j,\ell}\}\cup\Omega^{\mathrm{LL}}\cup\Omega^{\mathrm{DV}}}{\mathbf{Maximise}} \quad (4a)$$

$$\textbf{s.t.} \quad (4b) - (4c)$$

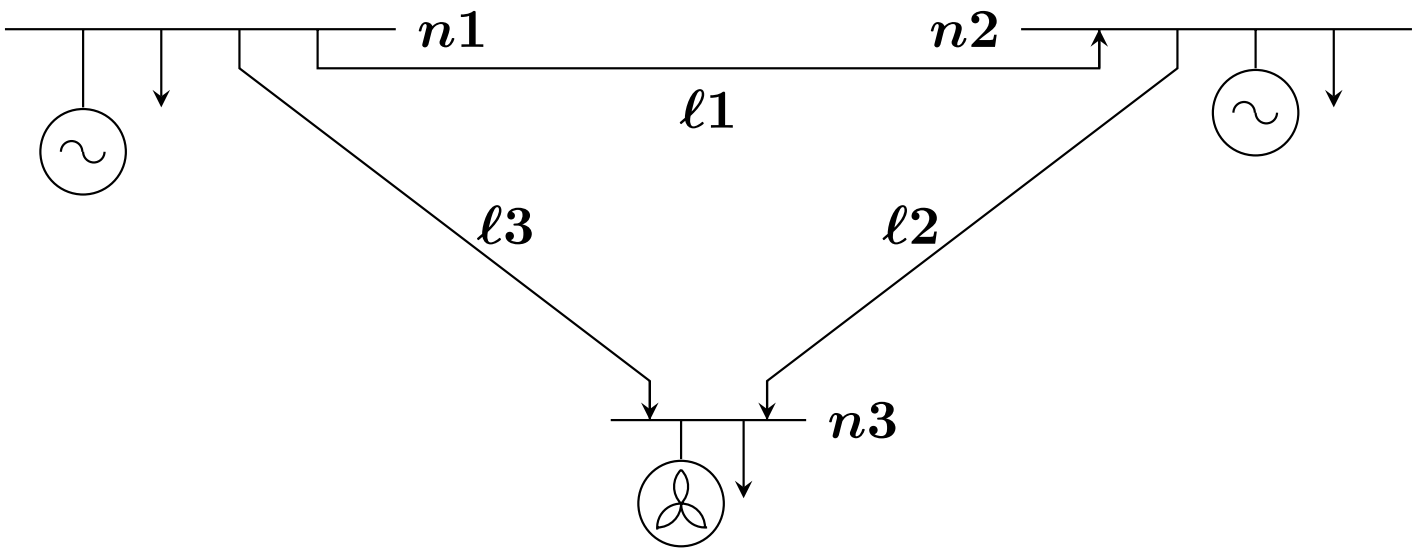
$$(2c) - (2d), (5d) - (6n)$$

$$(8a) - (8e), (8g) - (9j), (10a) - (10d)$$

$$(7a)$$

Numerical Examples

Three-Node Test Network



Demand and Generation Data

Parameter \ Node	$n1$	$n2$	$n3$
$A_{m,n,t}$	200	175	125
$Z_{m,n,t}$	1	1	1

Technology \ Attribute	F_u	C_u^{opr}	C_u^{gen}	$R_u^{\text{up}} = R_u^{\text{down}}$
$u1$	0.9	20	54.44	0.2
$u2$	0.5	35	16.06	0.5
$u3$	0	0	18.74	1

Week \ Period	$t1$	$t2$
$m1$	0.10	0.05
$m2$	0.15	0.25

Transmission Data

Level \ Attribute	$B_{j,\ell}$	$K^{\mathrm{trn}}_{j,\ell}$	$C^{\mathrm{trn}}_{j,\ell}$
$j1$	0	0	0
$j2$	1,200	3.7	23.8
$j3$	1,500	6.1	39.6
$j4$	1,700	12.2	79.4
$j5$	2,000	18.3	119.0
$j6$	2,300	24.4	158.6
$j7$	2,800	30.5	198.2
$j8$	3,600	36.6	238.0
$j9$	4,900	42.7	277.6
$j10$	5,100	48.8	317.2

Numerical Results for Three-Node PC Instances with $H = 1$

Metric \ D	0	25	50	75	100
SW	46.72	36.45	28.84	24.37	20.98
CS	46.12	34.29	25.83	19.76	16.80
PS	0	0	0	0	0
MS	1.08	2.79	3.92	5.29	5.10
GR	0	9.09	10.86	10.91	12.31
DC	0	9.09	10.86	10.91	12.31
TP	0.48	0.63	0.91	0.67	0.91
EM (kt)	0.50	0.36	0.22	0.15	0.12
GC (MW)	[177 181 0]	[81 217 45]	[25 173 487]	[0 145 570]	[0 135 634]
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CS	46.12	43.79	43.67	43.27	43.27
PS	0	0	0	0	0
MS	1.08	3.05	2.83	0.41	0.41
GR	0	0	0	0	0
DC	0	10.42	20.53	27.60	36.79
TP	0.48	0.63	0.60	0.32	0.32
EM (kt)	0.50	0.42	0.41	0.37	0.37
GC (MW)	[177 181 0]	[104 230 45]	[104 223 80]	[104 181 320]	[104 181 320]
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SW	46.72	36.28	27.68	22.58	18.75
CS	46.12	38.88	33.93	29.61	25.80
PS	0	0	0	0	-0.10
MS	1.08	2.91	1.10	2.31	3.39
GR	0	4.88	6.68	8.67	9.68
DC	0	9.76	13.35	17.33	19.35
TP	0.48	0.63	0.67	0.67	0.66
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Numerical Results for Three-Node CO Instances with $H = 0$

Metric \ D	0	25	50	75	100
SW	32.51	27.58	22.85	18.22	13.59
CS	10.90	11.19	12.02	12.02	12.02
PS	21.54	21.27	20.34	20.34	20.34
MS	0.10	0.12	0.03	0.03	0.03
GR	0	0	0	0	0
DC	0	4.90	9.26	13.90	18.53
TP	0.04	0.10	0.28	0.28	0.28
EM (kt)	0.20	0.20	0.19	0.19	0.19
GC (MW)	[73 69 160]	[69 72 171]	[65 69 228]	[65 69 228]	[65 69 228]
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PS	21.54	18.39	15.79	13.97	12.59
MS	0.10	0.29	0.34	0.09	0.16
GR	0	2.21	3.82	4.96	5.76
DC	0	4.42	7.65	9.92	11.52
TP	0.04	0.16	0.32	0.40	0.48
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MS	0.10	0.29	0.34	0.09	0.16
GR	0	2.21	3.82	4.96	5.76
DC	0	4.42	7.65	9.92	11.52
TP	0.04	0.16	0.32	0.40	0.48
EM (kt)	0.20	0.18	0.15	0.13	0.12
GC (MW)	[73 69 160]	[60 69 194]	[49 64 244]	[41 59 279]	[33 58 293]
TC (MW)	[6.1 0 0]	[12.2 0 12.2]	[12.2 0 36.6]	[0 12.2 48.8]	[12.2 12.2 48.8]

Numerical Results for Three-Node CO Instances with $H = 0.5$

Metric \ D	0	25	50	75	100
SW	32.51	26.65	21.86	17.95	14.87
CS	10.90	10.34	9.88	9.25	8.35
PS	21.54	18.39	15.79	13.97	12.59
MS	0.10	0.29	0.34	0.09	0.16
GR	0	2.21	3.82	4.96	5.76
DC	0	4.42	7.65	9.92	11.52
TP	0.04	0.16	0.32	0.40	0.48
EM (kt)	0.20	0.18	0.15	0.13	0.12
GC (MW)	[73 69 160]	[60 69 194]	[49 64 244]	[41 59 279]	[33 58 293]
TC (MW)	[6.1 0 0]	[12.2 0 12.2]	[12.2 0 36.6]	[0 12.2 48.8]	[12.2 12.2 48.8]

Numerical Results for Three-Node CO Instances with $H = 0.5$

Metric \ D	0	25	50	75	100
SW	32.51	26.65	21.86	17.95	14.87
CS	10.90	10.34	9.88	9.25	8.35
PS	21.54	18.39	15.79	13.97	12.59
MS	0.10	0.29	0.34	0.09	0.16
GR	0	2.21	3.82	4.96	5.76
DC	0	4.42	7.65	9.92	11.52
TP	0.04	0.16	0.32	0.40	0.48
EM (kt)	0.20	0.18	0.15	0.13	0.12
GC (MW)	[73 69 160]	[60 69 194]	[49 64 244]	[41 59 279]	[33 58 293]
TC (MW)	[6.1 0 0]	[12.2 0 12.2]	[12.2 0 36.6]	[0 12.2 48.8]	[12.2 12.2 48.8]

Numerical Results for Three-Node CO Instances with $H = 1$

Metric \ D	0	25	50	75	100
SW	32.51	25.74	20.74	17.09	14.87
CS	10.90	9.88	8.22	6.38	5.02
PS	21.54	15.91	12.58	10.33	9.36
MS	0.10	0.27	0.37	0.86	1.36
GR	0	3.80	5.77	6.48	5.23
DC	0	3.80	5.77	6.48	5.23
TP	0.04	0.32	0.44	0.48	0.87
EM (kt)	0.20	0.15	0.12	0.09	0.05
GC (MW)	[73 69 160]	[48 65 247]	[32 58 290]	[18 54 299]	[4 48 349]
TC (MW)	[6.1 0 0]	[12.2 6.1 30.5]	[12.2 6.1 48.8]	[18.3 6.1 48.8]	[42.7 42.7 48.8]

Numerical Results for Three-Node CO Instances with $H = 1$

Metric \ D	0	25	50	75	100
SW	32.51	25.74	20.74	17.09	14.87
CS	10.90	9.88	8.22	6.38	5.02
PS	21.54	15.91	12.58	10.33	9.36
MS	0.10	0.27	0.37	0.86	1.36
GR	0	3.80	5.77	6.48	5.23
DC	0	3.80	5.77	6.48	5.23
TP	0.04	0.32	0.44	0.48	0.87
EM (kt)	0.20	0.15	0.12	0.09	0.05
GC (MW)	[73 69 160]	[48 65 247]	[32 58 290]	[18 54 299]	[4 48 349]
TC (MW)	[6.1 0 0]	[12.2 6.1 30.5]	[12.2 6.1 48.8]	[18.3 6.1 48.8]	[42.7 42.7 48.8]

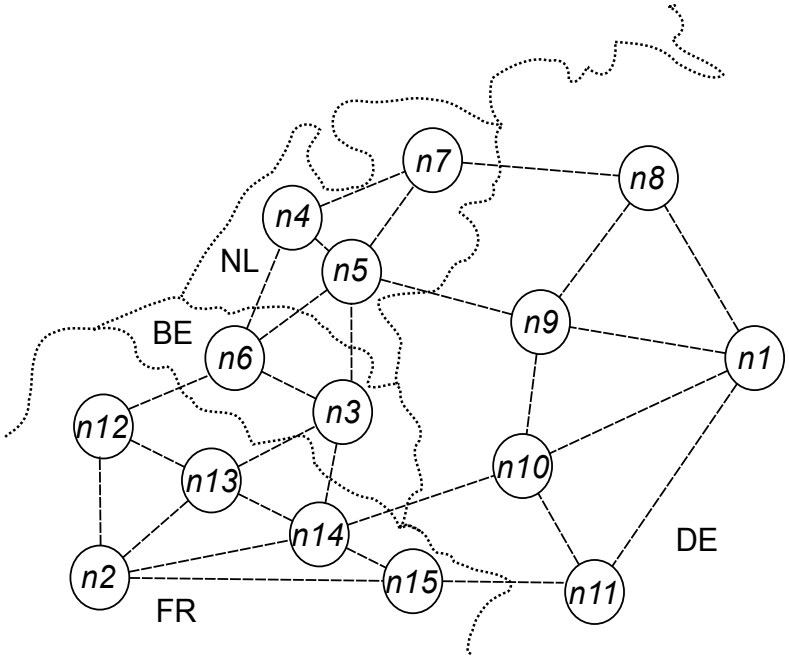
Numerical Results for Three-Node CO Instances with $H = 1$

Metric \ D	0	25	50	75	100
SW	32.51	25.74	20.74	17.09	14.87
CS	10.90	9.88	8.22	6.38	5.02
PS	21.54	15.91	12.58	10.33	9.36
MS	0.10	0.27	0.37	0.86	1.36
GR	0	3.80	5.77	6.48	5.23
DC	0	3.80	5.77	6.48	5.23
TP	0.04	0.32	0.44	0.48	0.87
EM (kt)	0.20	0.15	0.12	0.09	0.05
GC (MW)	[73 69 160]	[48 65 247]	[32 58 290]	[18 54 299]	[4 48 349]
TC (MW)	[6.1 0 0]	[12.2 6.1 30.5]	[12.2 6.1 48.8]	[18.3 6.1 48.8]	[42.7 42.7 48.8]

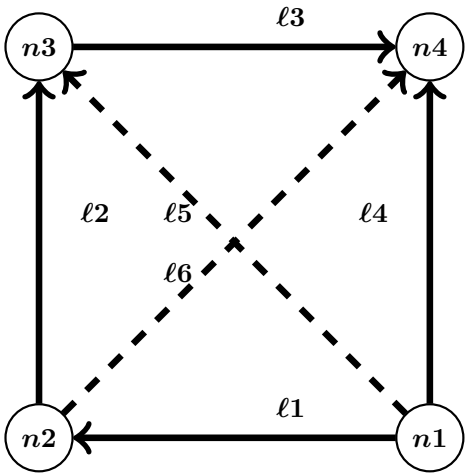
Numerical Results for Three-Node CO Instances with $H = 1$

Metric \ D	0	25	50	75	100
SW	32.51	25.74	20.74	17.09	14.87
CS	10.90	9.88	8.22	6.38	5.02
PS	21.54	15.91	12.58	10.33	9.36
MS	0.10	0.27	0.37	0.86	1.36
GR	0	3.80	5.77	6.48	5.23
DC	0	3.80	5.77	6.48	5.23
TP	0.04	0.32	0.44	0.48	0.87
EM (kt)	0.20	0.15	0.12	0.09	0.05
GC (MW)	[73 69 160]	[48 65 247]	[32 58 290]	[18 54 299]	[4 48 349]
TC (MW)	[6.1 0 0]	[12.2 6.1 30.5]	[12.2 6.1 48.8]	[18.3 6.1 48.8]	[42.7 42.7 48.8]

Stylised Western European Test Network



Neuhoff et al. (2005)



Data

All demand and generation data are for the year 2017 based on Virasjoki et al. (2020)

Level and Line \ Attribute	$B_{j,\ell}$	$K^{\text{trn}}_{j,\ell}$	$C^{\text{trn}}_{j,\ell}$
$j1$ and $\ell1$	567	2,608	0
$j1$ and $\ell2$	1,404	2,372	0
$j1$ and $\ell3$	2,202	2,218	0
$j1$ and $\ell4$	845	3,867	0
$j1$ and $\ell5$	0	0	0
$j1$ and $\ell6$	0	0	0
$j2$ and $\ell1$	680	3,008	840,000
$j2$ and $\ell2$	1,727	2,772	840,000
$j2$ and $\ell3$	2,730	2,618	840,000
$j2$ and $\ell4$	1,027	4,267	840,000
$j2$ and $\ell5$	220	400	1,260,000
$j2$ and $\ell6$	138	400	1,600,000

Numerical Results for PC Case Study with $H = 1$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,191.36	2,120.04	2,085.57	2,063.16
CS	1,902.62	1,729.18	1,579.70	1,509.93	1,464.83
PS	393.67	445.57	514.11	544.71	563.59
MS	10.03	18.29	27.92	33.46	38.93
GR	0	93.35	92.35	80.13	75.04
DC	0	93.35	92.35	80.13	75.04
TP	0	1.68	1.68	2.52	4.20
EM (Mt)	5.93	3.73	1.85	1.07	0.75
GC (GW)	[0 0 0 3.69]	[0 0 0 23.35]	[17.04 0 0 26.53]	[38.98 0 0 28.39]	[48.74 0 0 29.32]
TC (–)	[0 0 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 1 0 0]	[1 1 0 1 0 1]

Numerical Results for PC Case Study with $H = 1$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,191.36	2,120.04	2,085.57	2,063.16
CS	1,902.62	1,729.18	1,579.70	1,509.93	1,464.83
PS	393.67	445.57	514.11	544.71	563.59
MS	10.03	18.29	27.92	33.46	38.93
GR	0	93.35	92.35	80.13	75.04
DC	0	93.35	92.35	80.13	75.04
TP	0	1.68	1.68	2.52	4.20
EM (Mt)	5.93	3.73	1.85	1.07	0.75
GC (GW)	[0 0 0 3.69]	[0 0 0 23.35]	[17.04 0 0 26.53]	[38.98 0 0 28.39]	[48.74 0 0 29.32]
TC (–)	[0 0 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 1 0 0]	[1 1 0 1 0 1]

Numerical Results for PC Case Study with $H = 1$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,191.36	2,120.04	2,085.57	2,063.16
CS	1,902.62	1,729.18	1,579.70	1,509.93	1,464.83
PS	393.67	445.57	514.11	544.71	563.59
MS	10.03	18.29	27.92	33.46	38.93
GR	0	93.35	92.35	80.13	75.04
DC	0	93.35	92.35	80.13	75.04
TP	0	1.68	1.68	2.52	4.20
EM (Mt)	5.93	3.73	1.85	1.07	0.75
GC (GW)	[0 0 0 3.69]	[0 0 0 23.35]	[17.04 0 0 26.53]	[38.98 0 0 28.39]	[48.74 0 0 29.32]
TC (–)	[0 0 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 1 0 0]	[1 1 0 1 0 1]

Numerical Results for PC Case Study with $H = 1$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,191.36	2,120.04	2,085.57	2,063.16
CS	1,902.62	1,729.18	1,579.70	1,509.93	1,464.83
PS	393.67	445.57	514.11	544.71	563.59
MS	10.03	18.29	27.92	33.46	38.93
GR	0	93.35	92.35	80.13	75.04
DC	0	93.35	92.35	80.13	75.04
TP	0	1.68	1.68	2.52	4.20
EM (Mt)	5.93	3.73	1.85	1.07	0.75
GC (GW)	[0 0 0 3.69]	[0 0 0 23.35]	[17.04 0 0 26.53]	[38.98 0 0 28.39]	[48.74 0 0 29.32]
TC (–)	[0 0 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 1 0 0]	[1 1 0 1 0 1]

Numerical Results for PC Case Study with $H = 0$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,158.32	2,010.54	1,863.16	1,716.53
CS	1,902.62	1,901.98	1,898.53	1,909.38	1,909.38
PS	393.67	394.81	398.74	386.25	386.25
MS	10.03	10.21	10.50	10.35	10.35
GR	0	0	0	0	0
DC	0	147.84	295.54	439.87	586.50
TP	0	0.84	1.68	2.94	2.94
EM (Mt)	5.93	5.91	5.91	5.86	5.86
GC (GW)	[0 0 0 3.69]	[0 0 0 3.74]	[0 0 0 3.30]	[0 0 0 7.21]	[0 0 0 7.21]
TC (–)	[0 0 0 0 0 0]	[1 0 0 0 0 0]	[1 1 0 0 0 0]	[1 0 1 0 1 0]	[1 0 1 0 1 0]

Numerical Results for PC Case Study with $H = 0$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,158.32	2,010.54	1,863.16	1,716.53
CS	1,902.62	1,901.98	1,898.53	1,909.38	1,909.38
PS	393.67	394.81	398.74	386.25	386.25
MS	10.03	10.21	10.50	10.35	10.35
GR	0	0	0	0	0
DC	0	147.84	295.54	439.87	586.50
TP	0	0.84	1.68	2.94	2.94
EM (Mt)	5.93	5.91	5.91	5.86	5.86
GC (GW)	[0 0 0 3.69]	[0 0 0 3.74]	[0 0 0 3.30]	[0 0 0 7.21]	[0 0 0 7.21]
TC (–)	[0 0 0 0 0 0]	[1 0 0 0 0 0]	[1 1 0 0 0 0]	[1 0 1 0 1 0]	[1 0 1 0 1 0]

Numerical Results for PC Case Study with $H = 0$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,158.32	2,010.54	1,863.16	1,716.53
CS	1,902.62	1,901.98	1,898.53	1,909.38	1,909.38
PS	393.67	394.81	398.74	386.25	386.25
MS	10.03	10.21	10.50	10.35	10.35
GR	0	0	0	0	0
DC	0	147.84	295.54	439.87	586.50
TP	0	0.84	1.68	2.94	2.94
EM (Mt)	5.93	5.91	5.91	5.86	5.86
GC (GW)	[0 0 0 3.69]	[0 0 0 3.74]	[0 0 0 3.30]	[0 0 0 7.21]	[0 0 0 7.21]
TC (–)	[0 0 0 0 0 0]	[1 0 0 0 0 0]	[1 1 0 0 0 0]	[1 0 1 0 1 0]	[1 0 1 0 1 0]

Numerical Results for PC Case Study with $H = 0$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,158.32	2,010.54	1,863.16	1,716.53
CS	1,902.62	1,901.98	1,898.53	1,909.38	1,909.38
PS	393.67	394.81	398.74	386.25	386.25
MS	10.03	10.21	10.50	10.35	10.35
GR	0	0	0	0	0
DC	0	147.84	295.54	439.87	586.50
TP	0	0.84	1.68	2.94	2.94
EM (Mt)	5.93	5.91	5.91	5.86	5.86
GC (GW)	[0 0 0 3.69]	[0 0 0 3.74]	[0 0 0 3.30]	[0 0 0 7.21]	[0 0 0 7.21]
TC (–)	[0 0 0 0 0 0]	[1 0 0 0 0 0]	[1 1 0 0 0 0]	[1 0 1 0 1 0]	[1 0 1 0 1 0]

Numerical Results for PC Case Study with $H = 0.5$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,187.97	2,098.01	2,047.77	2,028.63
CS	1,902.62	1,812.45	1,729.18	1,647.48	1,579.02
PS	393.67	416.22	445.57	479.69	515.98
MS	10.03	14.18	18.29	23.73	29.56
GR	0	53.21	93.35	100.60	90.74
DC	0	106.41	186.70	201.21	181.48
TP	0	1.68	1.68	2.52	4.20
EM (Mt)	5.93	4.26	3.73	2.68	1.81
GC (GW)	[0 0 0 3.69]	[0 0 0 19.77]	[0 0 0 23.35]	[0 0 0 25.99]	[16.61 0 0 27.34]
TC (–)	[0 0 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 1 0 0]	[1 1 0 1 0 1]

Numerical Results for PC Case Study with $H = 0.5$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,187.97	2,098.01	2,047.77	2,028.63
CS	1,902.62	1,812.45	1,729.18	1,647.48	1,579.02
PS	393.67	416.22	445.57	479.69	515.98
MS	10.03	14.18	18.29	23.73	29.56
GR	0	53.21	93.35	100.60	90.74
DC	0	106.41	186.70	201.21	181.48
TP	0	1.68	1.68	2.52	4.20
EM (Mt)	5.93	4.26	3.73	2.68	1.81
GC (GW)	[0 0 0 3.69]	[0 0 0 19.77]	[0 0 0 23.35]	[0 0 0 25.99]	[16.61 0 0 27.34]
TC (–)	[0 0 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 1 0 0]	[1 1 0 1 0 1]

Numerical Results for PC Case Study with $H = 0.5$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,187.97	2,098.01	2,047.77	2,028.63
CS	1,902.62	1,812.45	1,729.18	1,647.48	1,579.02
PS	393.67	416.22	445.57	479.69	515.98
MS	10.03	14.18	18.29	23.73	29.56
GR	0	53.21	93.35	100.60	90.74
DC	0	106.41	186.70	201.21	181.48
TP	0	1.68	1.68	2.52	4.20
EM (Mt)	5.93	4.26	3.73	2.68	1.81
GC (GW)	[0 0 0 3.69]	[0 0 0 19.77]	[0 0 0 23.35]	[0 0 0 25.99]	[16.61 0 0 27.34]
TC (–)	[0 0 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 1 0 0]	[1 1 0 1 0 1]

Numerical Results for PC Case Study with $H = 0.5$

Metric \ D	0	25	50	75	100
SW	2,306.32	2,187.97	2,098.01	2,047.77	2,028.63
CS	1,902.62	1,812.45	1,729.18	1,647.48	1,579.02
PS	393.67	416.22	445.57	479.69	515.98
MS	10.03	14.18	18.29	23.73	29.56
GR	0	53.21	93.35	100.60	90.74
DC	0	106.41	186.70	201.21	181.48
TP	0	1.68	1.68	2.52	4.20
EM (Mt)	5.93	4.26	3.73	2.68	1.81
GC (GW)	[0 0 0 3.69]	[0 0 0 19.77]	[0 0 0 23.35]	[0 0 0 25.99]	[16.61 0 0 27.34]
TC (–)	[0 0 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 0 0 0]	[1 1 0 1 0 0]	[1 1 0 1 0 1]

Numerical Results for CO Case Study with $H = 0$

Metric \ D	0	25	50	75	100
SW	2,061.28	1,865.56	1,670.11	1,474.91	1,280.34
CS	1,709.65	1,709.65	1,709.80	1,713.06	1,713.06
PS	347.05	347.05	346.92	342.71	342.71
MS	4.58	4.58	4.89	4.95	4.95
GR	0	0	0	0	0
DC	0	195.71	390.65	583.71	778.28
TP	0	0	0.84	2.10	2.10
EM (Mt)	7.83	7.83	7.81	7.78	7.78
GC (GW)	[0 38.26 6.68 15.26]	[0 38.26 6.68 15.26]	[0 38.23 6.45 15.93]	[0 38.68 6.53 16.74]	[0 38.68 6.53 16.74]
TC (–)	[0 0 0 0 0 0]	[0 0 0 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 1 0]	[0 0 1 0 1 0]

Numerical Results for CO Case Study with $H = 0$

Metric \ D	0	25	50	75	100
SW	2,061.28	1,865.56	1,670.11	1,474.91	1,280.34
CS	1,709.65	1,709.65	1,709.80	1,713.06	1,713.06
PS	347.05	347.05	346.92	342.71	342.71
MS	4.58	4.58	4.89	4.95	4.95
GR	0	0	0	0	0
DC	0	195.71	390.65	583.71	778.28
TP	0	0	0.84	2.10	2.10
EM (Mt)	7.83	7.83	7.81	7.78	7.78
GC (GW)	[0 38.26 6.68 15.26]	[0 38.26 6.68 15.26]	[0 38.23 6.45 15.93]	[0 38.68 6.53 16.74]	[0 38.68 6.53 16.74]
TC (–)	[0 0 0 0 0 0]	[0 0 0 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 1 0]	[0 0 1 0 1 0]

Numerical Results for CO Case Study with $H = 0$

Metric \ D	0	25	50	75	100
SW	2,061.28	1,865.56	1,670.11	1,474.91	1,280.34
CS	1,709.65	1,709.65	1,709.80	1,713.06	1,713.06
PS	347.05	347.05	346.92	342.71	342.71
MS	4.58	4.58	4.89	4.95	4.95
GR	0	0	0	0	0
DC	0	195.71	390.65	583.71	778.28
TP	0	0	0.84	2.10	2.10
EM (Mt)	7.83	7.83	7.81	7.78	7.78
GC (GW)	[0 38.26 6.68 15.26]	[0 38.26 6.68 15.26]	[0 38.23 6.45 15.93]	[0 38.68 6.53 16.74]	[0 38.68 6.53 16.74]
TC (–)	[0 0 0 0 0 0]	[0 0 0 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 1 0]	[0 0 1 0 1 0]

Numerical Results for CO Case Study with $H = 0$

Metric \ D	0	25	50	75	100
SW	2,061.28	1,865.56	1,670.11	1,474.91	1,280.34
CS	1,709.65	1,709.65	1,709.80	1,713.06	1,713.06
PS	347.05	347.05	346.92	342.71	342.71
MS	4.58	4.58	4.89	4.95	4.95
GR	0	0	0	0	0
DC	0	195.71	390.65	583.71	778.28
TP	0	0	0.84	2.10	2.10
EM (Mt)	7.83	7.83	7.81	7.78	7.78
GC (GW)	[0 38.26 6.68 15.26]	[0 38.26 6.68 15.26]	[0 38.23 6.45 15.93]	[0 38.68 6.53 16.74]	[0 38.68 6.53 16.74]
TC (–)	[0 0 0 0 0 0]	[0 0 0 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 1 0]	[0 0 1 0 1 0]

Numerical Results for CO Case Study with $H = 0.5$

Metric \ D	0	25	50	75	100
SW	2,061.28	1,891.48	1,763.51	1,697.45	1,687.66
CS	1,709.65	1,608.20	1,502.59	1,413.73	1,357.27
PS	347.05	358.79	391.98	424.42	450.10
MS	4.58	6.16	9.62	11.69	13.28
GR	0	81.67	139.83	151.55	131.31
DC	0	163.34	279.66	303.10	262.63
TP	0	0	0.84	0.84	1.68
EM (Mt)	7.83	6.53	5.59	4.04	2.63
GC (GW)	[0 38.26 6.68 15.26]	[0 36.15 6.32 21.88]	[0 34.05 5.66 25.40]	[10.97 63.05 5.05 26.94]	[30.73 98.97 4.50 28.42]
TC (–)	[0 0 0 0 0 0]	[0 0 0 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 0 0]	[0 0 1 1 0 0]

Numerical Results for CO Case Study with $H = 0.5$

Metric \ D	0	25	50	75	100
SW	2,061.28	1,891.48	1,763.51	1,697.45	1,687.66
CS	1,709.65	1,608.20	1,502.59	1,413.73	1,357.27
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TC (–)	[0 0 0 0 0 0]	[0 0 0 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 0 0]	[0 0 1 1 0 0]

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TC (–)	[0 0 0 0 0 0]	[0 0 0 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 0 0]	[0 0 1 1 0 0]

Numerical Results for CO Case Study with $H = 1$

Metric \ D	0	25	50	75	100
SW	2,061.28	1,903.46	1,819.22	1,795.43	1,786.41
CS	1,709.65	1,502.53	1,356.92	1,268.09	1,212.11
PS	347.05	391.98	450.32	511.06	553.41
MS	4.58	8.96	12.83	17.11	21.73
GR	0	140.18	131.80	108.70	83.33
DC	0	140.18	131.80	108.70	83.33
TP	0	0	0.84	0.84	1.68
EM (Mt)	7.83	5.61	2.64	1.45	0.83
GC (GW)	[0 38.26 6.68 15.26]	[0 34.07 5.92 24.86]	[31.62 98.51 4.52 27.75]	[49.46 114.61 3.63 28.67]	[59.60 124.49 2.49 29.24]
TC (–)	[0 0 0 0 0 0]	[0 0 0 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 0 0]

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TC (–)	[0 0 0 0 0 0]	[0 0 0 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 0 0]	[0 0 1 0 0 0]

Conclusions

Summary

- Game-theoretic approach to compare CP, PC, and CO settings in analysing sustainable transmission expansion
  - CP matches the most efficient resource with demand
  - PC: lack of curb on consumption leads to smaller lines, but a carbon price internalises the damage cost and increases transmission capacity
  - CO: firms' market power boosts fossil-fuelled capacity, and a carbon price increases (decreases) transmission capacity with a greenfield (brownfield) assumption
  - A full carbon price results in perfect alignment of incentives under PC
  - Full carbon pricing may worsen outcomes under CO with a greenfield assumption: although CO_2 emissions are considerably lower, the curb on consumption is severe
- Future work: stochastic model, endogenous carbon pricing, improved numerical resolution methods

Appendix

CP MIQP Formulation

$$\underset{\Omega^{\mathrm{CP}}}{\text{Maximise}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} W_m \Bigg[\sum_{n\in\mathcal{N}}\left(A_{m,n,t}c_{m,n,t} - \frac{1}{2}Z_{m,n,t}c^2_{m,n,t}\right) - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} C^{\mathrm{sto}} r^{\mathrm{out}}_{i,m,n,t}$$

$$- \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\mathrm{opr}}_u y_{i,m,n,t,u}\Bigg] - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\mathrm{gen}}_u z_{i,n,u}$$

$$- \sum_{j\in\mathcal{J}_\ell}\sum_{\ell\in\mathcal{L}} C^{\mathrm{trn}}_{j,\ell} x_{j,\ell} - D\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}} W_m F_u y_{i,m,n,t,u} \tag{2a}$$

$$\text{s.t.} \sum_{j\in\mathcal{J}_\ell} x_{j,\ell} = 1,\ \forall \ell \tag{2b}$$

$$-\left(1-x_{j,\ell}\right)M^{\mathrm{trn}} \le f_{j,\ell,m,t} - B_{j,\ell}\left(v_{m,n^+_\ell,t} - v_{m,n^-_\ell,t}\right) \le \left(1-x_{j,\ell}\right)M^{\mathrm{trn}},$$

$$\forall \ell, m, t, j \in \mathcal{J}_\ell \tag{2c}$$

$$-T_t K^{\mathrm{trn}}_{j,\ell} x_{j,\ell} \le T_t f_{j,\ell,m,t} \le T_t K^{\mathrm{trn}}_{j,\ell} x_{j,\ell}, \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{2d}$$

$$\hat{f}_{\ell,m,t} = \sum_{j\in\mathcal{J}_\ell} f_{j,\ell,m,t}, \forall \ell, m, t \tag{2e}$$

$$-\pi \le v_{m,n,t} \le \pi,\ \forall m, n, t \tag{2f}$$

$$T_t \underline{G}_{m,n,t,u}\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right) \le y_{i,m,n,t,u} \le T_t \overline{G}_{m,n,t,u}\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right),$$

$$\forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2g}$$

$$y_{i,m,n,t,u} - y_{i,m,n,t-1,u} \le T_t R^{\mathrm{up}}_u\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right), \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2h}$$

$$y_{i,m,n,t-1,u} - y_{i,m,n,t,u} \le T_t R^{\mathrm{down}}_u\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right), \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2i}$$

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$$\underset{\Omega^{\mathrm{CP}}}{\text{Maximise}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} W_m \Bigg[\sum_{n\in\mathcal{N}}\left(A_{m,n,t}c_{m,n,t} - \frac{1}{2}Z_{m,n,t}c^2_{m,n,t}\right) - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} C^{\mathrm{sto}} r^{\mathrm{out}}_{i,m,n,t}$$

$$- \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\mathrm{opr}}_u y_{i,m,n,t,u}\Bigg] - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\mathrm{gen}}_u z_{i,n,u}$$

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$$\forall \ell, m, t, j \in \mathcal{J}_\ell \tag{2c}$$

$$-T_t K^{\mathrm{trn}}_{j,\ell} x_{j,\ell} \le T_t f_{j,\ell,m,t} \le T_t K^{\mathrm{trn}}_{j,\ell} x_{j,\ell}, \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{2d}$$

$$\hat{f}_{\ell,m,t} = \sum_{j\in\mathcal{J}_\ell} f_{j,\ell,m,t}, \forall \ell, m, t \tag{2e}$$

$$-\pi \le v_{m,n,t} \le \pi,\ \forall m, n, t \tag{2f}$$

$$T_t \underline{G}_{m,n,t,u}\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right) \le y_{i,m,n,t,u} \le T_t \overline{G}_{m,n,t,u}\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right),$$

$$\forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2g}$$

$$y_{i,m,n,t,u} - y_{i,m,n,t-1,u} \le T_t R^{\mathrm{up}}_u\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right), \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2h}$$

$$y_{i,m,n,t-1,u} - y_{i,m,n,t,u} \le T_t R^{\mathrm{down}}_u\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right), \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2i}$$

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$$T_t \underline{G}_{m,n,t,u}\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right) \le y_{i,m,n,t,u} \le T_t \overline{G}_{m,n,t,u}\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right),$$

$$\forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2g}$$

$$y_{i,m,n,t,u} - y_{i,m,n,t-1,u} \le T_t R^{\mathrm{up}}_u\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right), \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2h}$$

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$$\forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2g}$$

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$$T_t \underline{G}_{m,n,t,u}\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right) \le y_{i,m,n,t,u} \le T_t \overline{G}_{m,n,t,u}\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right),$$

$$\forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2g}$$

$$y_{i,m,n,t,u} - y_{i,m,n,t-1,u} \le T_t R^{\mathrm{up}}_u\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right), \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2h}$$

$$y_{i,m,n,t-1,u} - y_{i,m,n,t,u} \le T_t R^{\mathrm{down}}_u\left(K^{\mathrm{gen}}_{i,n,u} + z_{i,n,u}\right), \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{2i}$$

CP MIQP Formulation (cont'd)

$$r^{\text{sto}}_{i,m,n,t} - \left(E^{\text{sto}}\right)^{T_t} r^{\text{sto}}_{i,m,n,t-1} - E^{\text{in}} r^{\text{in}}_{i,m,n,t} + r^{\text{out}}_{i,m,n,t} = 0,\ \forall i, m, n, t \tag{3a}$$

$$r^{\text{in}}_{i,m,n,t} \leq T_t R^{\text{in}} \overline{R}_{i,n},\ \forall i, m, n, t \tag{3b}$$

$$r^{\text{out}}_{i,m,n,t} \leq T_t R^{\text{out}} \overline{R}_{i,n},\ \forall i, m, n, t \tag{3c}$$

$$\underline{R}_{i,n} \overline{R}_{i,n} \leq r^{\text{sto}}_{i,m,n,t} \leq \overline{R}_{i,n},\ \forall i, m, n, t \tag{3d}$$

$$c_{m,n,t} - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{i,n}} y_{i,m,n,t,u} + V T_t \sum_{\ell \in \mathcal{L}^+_n} \hat{f}_{\ell,m,t} - V T_t \sum_{\ell \in \mathcal{L}^-_n} \hat{f}_{\ell,m,t} - \sum_{i \in \mathcal{I}} r^{\text{out}}_{i,m,n,t}$$

$$+ \sum_{i \in \mathcal{I}} r^{\text{in}}_{i,m,n,t} = 0,\ \forall m, n, t \tag{3e}$$

$$x_{j,\ell} \in \{0, 1\}, \forall \ell, j \in \mathcal{J}_\ell \tag{3f}$$

$$c_{m,n,t} \geq 0,\ \forall m, n, t \tag{3g}$$

$$y_{i,m,n,t,u} \geq 0,\ \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{3h}$$

$$z_{i,n,u} \geq 0,\ \forall i, n, u \in \mathcal{U}_{i,n} \tag{3i}$$

$$r^{\text{sto}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3j}$$

$$r^{\text{in}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3k}$$

$$r^{\text{out}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3l}$$

$$f_{j,\ell,m,t} \text{ u.r.s.},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{3m}$$

$$\hat{f}_{\ell,m,t} \text{ u.r.s.},\ \forall \ell, m, t \tag{3n}$$

$$v_{m,n,t} \text{ u.r.s.},\ \forall m, n, t \tag{3o}$$

CP MIQP Formulation (cont'd)

$$r^{\text{sto}}_{i,m,n,t} - \left(E^{\text{sto}}\right)^{T_t} r^{\text{sto}}_{i,m,n,t-1} - E^{\text{in}} r^{\text{in}}_{i,m,n,t} + r^{\text{out}}_{i,m,n,t} = 0,\ \forall i, m, n, t \tag{3a}$$

$$r^{\text{in}}_{i,m,n,t} \leq T_t R^{\text{in}} \overline{R}_{i,n},\ \forall i, m, n, t \tag{3b}$$

$$r^{\text{out}}_{i,m,n,t} \leq T_t R^{\text{out}} \overline{R}_{i,n},\ \forall i, m, n, t \tag{3c}$$

$$\underline{R}_{i,n} \overline{R}_{i,n} \leq r^{\text{sto}}_{i,m,n,t} \leq \overline{R}_{i,n},\ \forall i, m, n, t \tag{3d}$$

$$c_{m,n,t} - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{i,n}} y_{i,m,n,t,u} + V T_t \sum_{\ell \in \mathcal{L}^+_n} \hat{f}_{\ell,m,t} - V T_t \sum_{\ell \in \mathcal{L}^-_n} \hat{f}_{\ell,m,t} - \sum_{i \in \mathcal{I}} r^{\text{out}}_{i,m,n,t}$$

$$+ \sum_{i \in \mathcal{I}} r^{\text{in}}_{i,m,n,t} = 0,\ \forall m, n, t \tag{3e}$$

$$x_{j,\ell} \in \{0, 1\}, \forall \ell, j \in \mathcal{J}_\ell \tag{3f}$$

$$c_{m,n,t} \geq 0,\ \forall m, n, t \tag{3g}$$

$$y_{i,m,n,t,u} \geq 0,\ \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{3h}$$

$$z_{i,n,u} \geq 0,\ \forall i, n, u \in \mathcal{U}_{i,n} \tag{3i}$$

$$r^{\text{sto}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3j}$$

$$r^{\text{in}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3k}$$

$$r^{\text{out}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3l}$$

$$f_{j,\ell,m,t} \text{ u.r.s.},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{3m}$$

$$\hat{f}_{\ell,m,t} \text{ u.r.s.},\ \forall \ell, m, t \tag{3n}$$

$$v_{m,n,t} \text{ u.r.s.},\ \forall m, n, t \tag{3o}$$

CP MIQP Formulation (cont'd)

$$r^{\text{sto}}_{i,m,n,t} - \left(E^{\text{sto}}\right)^{T_t} r^{\text{sto}}_{i,m,n,t-1} - E^{\text{in}} r^{\text{in}}_{i,m,n,t} + r^{\text{out}}_{i,m,n,t} = 0,\ \forall i, m, n, t \tag{3a}$$

$$r^{\text{in}}_{i,m,n,t} \leq T_t R^{\text{in}} \overline{R}_{i,n},\ \forall i, m, n, t \tag{3b}$$

$$r^{\text{out}}_{i,m,n,t} \leq T_t R^{\text{out}} \overline{R}_{i,n},\ \forall i, m, n, t \tag{3c}$$

$$\underline{R}_{i,n} \overline{R}_{i,n} \leq r^{\text{sto}}_{i,m,n,t} \leq \overline{R}_{i,n},\ \forall i, m, n, t \tag{3d}$$

$$c_{m,n,t} - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{i,n}} y_{i,m,n,t,u} + V T_t \sum_{\ell \in \mathcal{L}^+_n} \hat{f}_{\ell,m,t} - V T_t \sum_{\ell \in \mathcal{L}^-_n} \hat{f}_{\ell,m,t} - \sum_{i \in \mathcal{I}} r^{\text{out}}_{i,m,n,t}$$

$$+ \sum_{i \in \mathcal{I}} r^{\text{in}}_{i,m,n,t} = 0,\ \forall m, n, t \tag{3e}$$

$$x_{j,\ell} \in \{0, 1\}, \forall \ell, j \in \mathcal{J}_\ell \tag{3f}$$

$$c_{m,n,t} \geq 0,\ \forall m, n, t \tag{3g}$$

$$y_{i,m,n,t,u} \geq 0,\ \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{3h}$$

$$z_{i,n,u} \geq 0,\ \forall i, n, u \in \mathcal{U}_{i,n} \tag{3i}$$

$$r^{\text{sto}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3j}$$

$$r^{\text{in}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3k}$$

$$r^{\text{out}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3l}$$

$$f_{j,\ell,m,t} \text{ u.r.s.},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{3m}$$

$$\hat{f}_{\ell,m,t} \text{ u.r.s.},\ \forall \ell, m, t \tag{3n}$$

$$v_{m,n,t} \text{ u.r.s.},\ \forall m, n, t \tag{3o}$$

CP MIQP Formulation (cont'd)

$$r^{\text{sto}}_{i,m,n,t} - \left(E^{\text{sto}}\right)^{T_t} r^{\text{sto}}_{i,m,n,t-1} - E^{\text{in}} r^{\text{in}}_{i,m,n,t} + r^{\text{out}}_{i,m,n,t} = 0,\ \forall i, m, n, t \tag{3a}$$

$$r^{\text{in}}_{i,m,n,t} \leq T_t R^{\text{in}} \overline{R}_{i,n},\ \forall i, m, n, t \tag{3b}$$

$$r^{\text{out}}_{i,m,n,t} \leq T_t R^{\text{out}} \overline{R}_{i,n},\ \forall i, m, n, t \tag{3c}$$

$$\underline{R}_{i,n} \overline{R}_{i,n} \leq r^{\text{sto}}_{i,m,n,t} \leq \overline{R}_{i,n},\ \forall i, m, n, t \tag{3d}$$

$$c_{m,n,t} - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{i,n}} y_{i,m,n,t,u} + V T_t \sum_{\ell \in \mathcal{L}^+_n} \hat{f}_{\ell,m,t} - V T_t \sum_{\ell \in \mathcal{L}^-_n} \hat{f}_{\ell,m,t} - \sum_{i \in \mathcal{I}} r^{\text{out}}_{i,m,n,t}$$

$$+ \sum_{i \in \mathcal{I}} r^{\text{in}}_{i,m,n,t} = 0,\ \forall m, n, t \tag{3e}$$

$$x_{j,\ell} \in \{0, 1\}, \forall \ell, j \in \mathcal{J}_\ell \tag{3f}$$

$$c_{m,n,t} \geq 0,\ \forall m, n, t \tag{3g}$$

$$y_{i,m,n,t,u} \geq 0,\ \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{3h}$$

$$z_{i,n,u} \geq 0,\ \forall i, n, u \in \mathcal{U}_{i,n} \tag{3i}$$

$$r^{\text{sto}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3j}$$

$$r^{\text{in}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3k}$$

$$r^{\text{out}}_{i,m,n,t} \geq 0,\ \forall i, m, n, t \tag{3l}$$

$$f_{j,\ell,m,t} \text{ u.r.s.},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{3m}$$

$$\hat{f}_{\ell,m,t} \text{ u.r.s.},\ \forall \ell, m, t \tag{3n}$$

$$v_{m,n,t} \text{ u.r.s.},\ \forall m, n, t \tag{3o}$$

Bi-Level Formulation: Upper Level

$$\underset{x_{j,\ell}}{\text{Maximise}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} W_m \left[\sum_{n\in\mathcal{N}}\left(A_{m,n,t}c_{m,n,t} - \frac{1}{2}Z_{m,n,t}c_{m,n,t}^2\right)\right.$$

$$\left. - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C_u^{\text{opr}} y_{i,m,n,t,u} - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} C^{\text{sto}} r_{i,m,n,t}^{\text{out}}\right]$$

$$- \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C_u^{\text{gen}} z_{i,n,u} - \sum_{j\in\mathcal{J}_\ell}\sum_{\ell\in\mathcal{L}} C_{j,\ell}^{\text{trn}} x_{j,\ell}$$

$$- D\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}} W_m F_u y_{i,m,n,t,u} \tag{4a}$$

$$\text{s.t. } x_{j,\ell} \in \{0,1\}, \forall \ell, j \in \mathcal{J}_\ell \tag{4b}$$

$$\sum_{j\in\mathcal{J}_\ell} x_{j,\ell} = 1, \forall \ell \tag{4c}$$

Bi-Level Formulation: Lower Level

$$\underset{\Omega^{LL}}{\text{Maximise}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} W_m \left[\sum_{n\in\mathcal{N}}\left(A_{m,n,t}c_{m,n,t} - \frac{1}{2}Z_{m,n,t}c^2_{m,n,t}\right) - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} C^{\text{sto}} r^{\text{out}}_{i,m,n,t}\right.$$

$$\left. - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{opr}}_u y_{i,m,n,t,u}\right] - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{gen}}_u z_{i,n,u}$$

$$- \frac{1}{2}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} W_m Z_{m,n,t}\sum_{i\in\mathcal{I}}\left(\sum_{u\in\mathcal{U}_{i,n}} y_{i,m,n,t,u} + r^{\text{out}}_{i,m,n,t} - r^{\text{in}}_{i,m,n,t}\right)^2$$

$$- HD\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}} W_m F_u y_{i,m,n,t,u} \tag{5a}$$

$$\text{s.t. } f_{j,\ell,m,t} = x_{j,\ell}B_{j,\ell}\left(v_{n^+_\ell,m,t} - v_{n^-_\ell,m,t}\right) : \mu_{j,\ell,m,t},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{5b}$$

$$\underline{\mu}_{j,\ell,m,t} : -T_t K^{\text{trn}}_{j,\ell} \le T_t f_{j,\ell,m,t} \le T_t K^{\text{trn}}_{j,\ell} : \overline{\mu}_{j,\ell,m,t},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{5c}$$

$$\hat{f}_{\ell,m,t} = \sum_{j\in\mathcal{J}_\ell} f_{j,\ell,m,t} : \psi_{\ell,m,t},\ \forall \ell, m, t \tag{5d}$$

$$\underline{\kappa}_{m,n,t} : -\pi \le v_{m,n,t} \le \pi : \overline{\kappa}_{m,n,t},\ \forall m, n, t \tag{5e}$$

$$\underline{\beta}_{i,m,n,t,u} : T_t \underline{G}_{m,n,t,u}\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) \le y_{i,m,n,t,u}$$

$$\le T_t \overline{G}_{m,n,t,u}\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \overline{\beta}_{i,m,n,t,u}, \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5f}$$

$$y_{i,m,n,t,u} - y_{i,m,n,t-1,u} \le T_t R^{\text{up}}_u\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \beta^{\text{up}}_{i,m,n,t,u}, \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5g}$$

$$y_{i,m,n,t-1,u} - y_{i,m,n,t,u} \le T_t R^{\text{down}}_u\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \beta^{\text{down}}_{i,m,n,t,u},$$

$$\forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5h}$$

Bi-Level Formulation: Lower Level

$$\underset{\Omega^{LL}}{\text{Maximise}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} W_m \left[\sum_{n\in\mathcal{N}}\left(A_{m,n,t}c_{m,n,t} - \frac{1}{2}Z_{m,n,t}c^2_{m,n,t}\right) - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} C^{\text{sto}} r^{\text{out}}_{i,m,n,t}\right.$$

$$\left. - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{opr}}_u y_{i,m,n,t,u}\right] - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{gen}}_u z_{i,n,u}$$

$$- \frac{1}{2}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} W_m Z_{m,n,t}\sum_{i\in\mathcal{I}}\left(\sum_{u\in\mathcal{U}_{i,n}} y_{i,m,n,t,u} + r^{\text{out}}_{i,m,n,t} - r^{\text{in}}_{i,m,n,t}\right)^2$$

$$- HD\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}} W_m F_u y_{i,m,n,t,u} \tag{5a}$$

s.t. $$f_{j,\ell,m,t} = x_{j,\ell}B_{j,\ell}\left(v_{n^+_\ell,m,t} - v_{n^-_\ell,m,t}\right) : \mu_{j,\ell,m,t},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{5b}$$

$$\underline{\mu}_{j,\ell,m,t} : -T_t K^{\text{trn}}_{j,\ell} \le T_t f_{j,\ell,m,t} \le T_t K^{\text{trn}}_{j,\ell} : \overline{\mu}_{j,\ell,m,t},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{5c}$$

$$\hat{f}_{\ell,m,t} = \sum_{j\in\mathcal{J}_\ell} f_{j,\ell,m,t} : \psi_{\ell,m,t},\ \forall \ell, m, t \tag{5d}$$

$$\underline{\kappa}_{m,n,t} : -\pi \le v_{m,n,t} \le \pi : \overline{\kappa}_{m,n,t},\ \forall m, n, t \tag{5e}$$

$$\underline{\beta}_{i,m,n,t,u} : T_t \underline{G}_{m,n,t,u}\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) \le y_{i,m,n,t,u}$$

$$\le T_t \overline{G}_{m,n,t,u}\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \overline{\beta}_{i,m,n,t,u}, \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5f}$$

$$y_{i,m,n,t,u} - y_{i,m,n,t-1,u} \le T_t R^{\text{up}}_u\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \beta^{\text{up}}_{i,m,n,t,u}, \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5g}$$

$$y_{i,m,n,t-1,u} - y_{i,m,n,t,u} \le T_t R^{\text{down}}_u\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \beta^{\text{down}}_{i,m,n,t,u},$$

$$\forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5h}$$

Bi-Level Formulation: Lower Level

$$\underset{\Omega^{LL}}{\text{Maximise}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} W_m \left[\sum_{n\in\mathcal{N}}\left(A_{m,n,t}c_{m,n,t} - \frac{1}{2}Z_{m,n,t}c^2_{m,n,t}\right) - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} C^{\text{sto}} r^{\text{out}}_{i,m,n,t}\right.$$

$$\left. - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{opr}}_u y_{i,m,n,t,u}\right] - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{gen}}_u z_{i,n,u}$$

$$- \frac{1}{2}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} W_m Z_{m,n,t}\sum_{i\in\mathcal{I}}\left(\sum_{u\in\mathcal{U}_{i,n}} y_{i,m,n,t,u} + r^{\text{out}}_{i,m,n,t} - r^{\text{in}}_{i,m,n,t}\right)^2$$

$$- HD\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}} W_m F_u y_{i,m,n,t,u} \tag{5a}$$

s.t. $$f_{j,\ell,m,t} = x_{j,\ell}B_{j,\ell}\left(v_{n^+_\ell,m,t} - v_{n^-_\ell,m,t}\right) : \mu_{j,\ell,m,t},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{5b}$$

$$\underline{\mu}_{j,\ell,m,t} : -T_t K^{\text{trn}}_{j,\ell} \le T_t f_{j,\ell,m,t} \le T_t K^{\text{trn}}_{j,\ell} : \overline{\mu}_{j,\ell,m,t},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{5c}$$

$$\hat{f}_{\ell,m,t} = \sum_{j\in\mathcal{J}_\ell} f_{j,\ell,m,t} : \psi_{\ell,m,t},\ \forall \ell, m, t \tag{5d}$$

$$\underline{\kappa}_{m,n,t} : -\pi \le v_{m,n,t} \le \pi : \overline{\kappa}_{m,n,t},\ \forall m, n, t \tag{5e}$$

$$\underline{\beta}_{i,m,n,t,u} : T_t \underline{G}_{m,n,t,u}\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) \le y_{i,m,n,t,u}$$

$$\le T_t \overline{G}_{m,n,t,u}\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \overline{\beta}_{i,m,n,t,u},\ \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5f}$$

$$y_{i,m,n,t,u} - y_{i,m,n,t-1,u} \le T_t R^{\text{up}}_u\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \beta^{\text{up}}_{i,m,n,t,u},\ \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5g}$$

$$y_{i,m,n,t-1,u} - y_{i,m,n,t,u} \le T_t R^{\text{down}}_u\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \beta^{\text{down}}_{i,m,n,t,u},$$

$$\forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5h}$$

Bi-Level Formulation: Lower Level

$$\underset{\Omega^{LL}}{\text{Maximise}} \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} W_m \left[\sum_{n\in\mathcal{N}}\left(A_{m,n,t}c_{m,n,t} - \frac{1}{2}Z_{m,n,t}c^2_{m,n,t}\right) - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} C^{\text{sto}} r^{\text{out}}_{i,m,n,t}\right.$$

$$\left. - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{opr}}_u y_{i,m,n,t,u}\right] - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{gen}}_u z_{i,n,u}$$

$$- \frac{1}{2}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} W_m Z_{m,n,t}\sum_{i\in\mathcal{I}}\left(\sum_{u\in\mathcal{U}_{i,n}} y_{i,m,n,t,u} + r^{\text{out}}_{i,m,n,t} - r^{\text{in}}_{i,m,n,t}\right)^2$$

$$- HD\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}} W_m F_u y_{i,m,n,t,u} \tag{5a}$$

$$\text{s.t. } f_{j,\ell,m,t} = x_{j,\ell}B_{j,\ell}\left(v_{n^+_\ell,m,t} - v_{n^-_\ell,m,t}\right) : \mu_{j,\ell,m,t},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{5b}$$

$$\underline{\mu}_{j,\ell,m,t} : -T_t K^{\text{trn}}_{j,\ell} \le T_t f_{j,\ell,m,t} \le T_t K^{\text{trn}}_{j,\ell} : \overline{\mu}_{j,\ell,m,t},\ \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{5c}$$

$$\hat{f}_{\ell,m,t} = \sum_{j\in\mathcal{J}_\ell} f_{j,\ell,m,t} : \psi_{\ell,m,t},\ \forall \ell, m, t \tag{5d}$$

$$\underline{\kappa}_{m,n,t} : -\pi \le v_{m,n,t} \le \pi : \overline{\kappa}_{m,n,t},\ \forall m, n, t \tag{5e}$$

$$\underline{\beta}_{i,m,n,t,u} : T_t \underline{G}_{m,n,t,u}\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) \le y_{i,m,n,t,u}$$

$$\le T_t \overline{G}_{m,n,t,u}\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \overline{\beta}_{i,m,n,t,u}, \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5f}$$

$$y_{i,m,n,t,u} - y_{i,m,n,t-1,u} \le T_t R^{\text{up}}_u\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \beta^{\text{up}}_{i,m,n,t,u}, \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5g}$$

$$y_{i,m,n,t-1,u} - y_{i,m,n,t,u} \le T_t R^{\text{down}}_u\left(K^{\text{gen}}_{i,n,u} + z_{i,n,u}\right) : \beta^{\text{down}}_{i,m,n,t,u},$$

$$\forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{5h}$$

Bi-Level Formulation: Lower Level (cont'd)

$$r^{\text{sto}}_{i,m,n,t} - \left(E^{\text{sto}}\right)^{T_t} r^{\text{sto}}_{i,m,n,t-1} - E^{\text{in}} r^{\text{in}}_{i,m,n,t} + r^{\text{out}}_{i,m,n,t} = 0 : \theta^{\text{sto}}_{i,m,n,t}, \forall i, m, n, t$$

(6a)

$$r^{\text{in}}_{i,m,n,t} \leq T_t R^{\text{in}} \overline{R}_{i,n} : \theta^{\text{in}}_{i,m,n,t}, \forall i, m, n, t \quad (6b)$$

$$r^{\text{out}}_{i,m,n,t} \leq T_t R^{\text{out}} \overline{R}_{i,n} : \theta^{\text{out}}_{i,m,n,t}, \forall i, m, n, t \quad (6c)$$

$$\underline{\theta}_{i,m,n,t} : \underline{R}_{i,n} \overline{R}_{i,n} \leq r^{\text{sto}}_{i,m,n,t} \leq \overline{R}_{i,n} : \overline{\theta}_{i,m,n,t}, \forall i, m, n, t \quad (6d)$$

$$c_{m,n,t} - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{i,n}} y_{i,m,n,t,u} + V T_t \sum_{\ell \in \mathcal{L}^+_n} \hat{f}_{\ell,m,t} - V T_t \sum_{\ell \in \mathcal{L}^-_n} \hat{f}_{\ell,m,t} - \sum_{i \in \mathcal{I}} r^{\text{out}}_{i,m,n,t}$$

$$+ \sum_{i \in \mathcal{I}} r^{\text{in}}_{i,m,n,t} = 0 : \lambda_{m,n,t}, \forall m, n, t \quad (6e)$$

$$c_{m,n,t} \geq 0 : \phi^{c}_{m,n,t}, \forall m, n, t \quad (6f)$$

$$y_{i,m,n,t,u} \geq 0 : \phi^{y}_{i,m,n,t,u}, \forall i, m, n, t, u \in \mathcal{U}_{i,n} \quad (6g)$$

$$z_{i,n,u} \geq 0 : \phi^{z}_{i,n,u}, \forall i, n, u \in \mathcal{U}_{i,n} \quad (6h)$$

$$r^{\text{sto}}_{i,m,n,t} \geq 0 : \phi^{\text{sto}}_{i,m,n,t}, \forall i, m, n, t \quad (6i)$$

$$r^{\text{in}}_{i,m,n,t} \geq 0 : \phi^{\text{in}}_{i,m,n,t}, \forall i, m, n, t \quad (6j)$$

$$r^{\text{out}}_{i,m,n,t} \geq 0 : \phi^{\text{out}}_{i,m,n,t}, \forall i, m, n, t \quad (6k)$$

$$f_{j,\ell,m,t} \text{ u.r.s., } \forall \ell, m, t, j \in \mathcal{J}_\ell \quad (6l)$$

$$\hat{f}_{\ell,m,t} \text{ u.r.s., } \forall \ell, m, t \quad (6m)$$

$$v_{m,n,t} \text{ u.r.s., } \forall m, n, t \quad (6n)$$

Bi-Level Formulation: Lower Level (cont'd)

$$r^{\text{sto}}_{i,m,n,t} - \left(E^{\text{sto}}\right)^{T_t} r^{\text{sto}}_{i,m,n,t-1} - E^{\text{in}} r^{\text{in}}_{i,m,n,t} + r^{\text{out}}_{i,m,n,t} = 0 : \theta^{\text{sto}}_{i,m,n,t}, \forall i, m, n, t$$

(6a)

$$r^{\text{in}}_{i,m,n,t} \leq T_t R^{\text{in}} \overline{R}_{i,n} : \theta^{\text{in}}_{i,m,n,t}, \forall i, m, n, t \tag{6b}$$

$$r^{\text{out}}_{i,m,n,t} \leq T_t R^{\text{out}} \overline{R}_{i,n} : \theta^{\text{out}}_{i,m,n,t}, \forall i, m, n, t \tag{6c}$$

$$\underline{\theta}_{i,m,n,t} : \underline{R}_{i,n} \overline{R}_{i,n} \leq r^{\text{sto}}_{i,m,n,t} \leq \overline{R}_{i,n} : \overline{\theta}_{i,m,n,t}, \forall i, m, n, t \tag{6d}$$

$$c_{m,n,t} - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{i,n}} y_{i,m,n,t,u} + V T_t \sum_{\ell \in \mathcal{L}^+_n} \hat{f}_{\ell,m,t} - V T_t \sum_{\ell \in \mathcal{L}^-_n} \hat{f}_{\ell,m,t} - \sum_{i \in \mathcal{I}} r^{\text{out}}_{i,m,n,t}$$

$$+ \sum_{i \in \mathcal{I}} r^{\text{in}}_{i,m,n,t} = 0 : \lambda_{m,n,t}, \forall m, n, t \tag{6e}$$

$$c_{m,n,t} \geq 0 : \phi^{c}_{m,n,t}, \forall m, n, t \tag{6f}$$

$$y_{i,m,n,t,u} \geq 0 : \phi^{y}_{i,m,n,t,u}, \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{6g}$$

$$z_{i,n,u} \geq 0 : \phi^{z}_{i,n,u}, \forall i, n, u \in \mathcal{U}_{i,n} \tag{6h}$$

$$r^{\text{sto}}_{i,m,n,t} \geq 0 : \phi^{\text{sto}}_{i,m,n,t}, \forall i, m, n, t \tag{6i}$$

$$r^{\text{in}}_{i,m,n,t} \geq 0 : \phi^{\text{in}}_{i,m,n,t}, \forall i, m, n, t \tag{6j}$$

$$r^{\text{out}}_{i,m,n,t} \geq 0 : \phi^{\text{out}}_{i,m,n,t}, \forall i, m, n, t \tag{6k}$$

$$f_{j,\ell,m,t} \text{ u.r.s., } \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{6l}$$

$$\hat{f}_{\ell,m,t} \text{ u.r.s., } \forall \ell, m, t \tag{6m}$$

$$v_{m,n,t} \text{ u.r.s., } \forall m, n, t \tag{6n}$$

Bi-Level Formulation: Lower Level (cont'd)

$$r^{\text{sto}}_{i,m,n,t} - \left(E^{\text{sto}}\right)^{T_t} r^{\text{sto}}_{i,m,n,t-1} - E^{\text{in}} r^{\text{in}}_{i,m,n,t} + r^{\text{out}}_{i,m,n,t} = 0 : \theta^{\text{sto}}_{i,m,n,t}, \forall i, m, n, t$$

(6a)

$$r^{\text{in}}_{i,m,n,t} \leq T_t R^{\text{in}} \overline{R}_{i,n} : \theta^{\text{in}}_{i,m,n,t}, \forall i, m, n, t \tag{6b}$$

$$r^{\text{out}}_{i,m,n,t} \leq T_t R^{\text{out}} \overline{R}_{i,n} : \theta^{\text{out}}_{i,m,n,t}, \forall i, m, n, t \tag{6c}$$

$$\underline{\theta}_{i,m,n,t} : \underline{R}_{i,n} \overline{R}_{i,n} \leq r^{\text{sto}}_{i,m,n,t} \leq \overline{R}_{i,n} : \overline{\theta}_{i,m,n,t}, \forall i, m, n, t \tag{6d}$$

$$c_{m,n,t} - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{i,n}} y_{i,m,n,t,u} + VT_t \sum_{\ell \in \mathcal{L}^+_n} \hat{f}_{\ell,m,t} - VT_t \sum_{\ell \in \mathcal{L}^-_n} \hat{f}_{\ell,m,t} - \sum_{i \in \mathcal{I}} r^{\text{out}}_{i,m,n,t}$$

$$+ \sum_{i \in \mathcal{I}} r^{\text{in}}_{i,m,n,t} = 0 : \lambda_{m,n,t}, \forall m, n, t \tag{6e}$$

$$c_{m,n,t} \geq 0 : \phi^{c}_{m,n,t}, \forall m, n, t \tag{6f}$$

$$y_{i,m,n,t,u} \geq 0 : \phi^{y}_{i,m,n,t,u}, \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{6g}$$

$$z_{i,n,u} \geq 0 : \phi^{z}_{i,n,u}, \forall i, n, u \in \mathcal{U}_{i,n} \tag{6h}$$

$$r^{\text{sto}}_{i,m,n,t} \geq 0 : \phi^{\text{sto}}_{i,m,n,t}, \forall i, m, n, t \tag{6i}$$

$$r^{\text{in}}_{i,m,n,t} \geq 0 : \phi^{\text{in}}_{i,m,n,t}, \forall i, m, n, t \tag{6j}$$

$$r^{\text{out}}_{i,m,n,t} \geq 0 : \phi^{\text{out}}_{i,m,n,t}, \forall i, m, n, t \tag{6k}$$

$$f_{j,\ell,m,t} \text{ u.r.s., } \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{6l}$$

$$\hat{f}_{\ell,m,t} \text{ u.r.s., } \forall \ell, m, t \tag{6m}$$

$$v_{m,n,t} \text{ u.r.s., } \forall m, n, t \tag{6n}$$

Bi-Level Formulation: Lower Level (cont'd)

$$r^{\text{sto}}_{i,m,n,t} - \left(E^{\text{sto}}\right)^{T_t} r^{\text{sto}}_{i,m,n,t-1} - E^{\text{in}} r^{\text{in}}_{i,m,n,t} + r^{\text{out}}_{i,m,n,t} = 0 : \theta^{\text{sto}}_{i,m,n,t}, \forall i, m, n, t \tag{6a}$$

$$r^{\text{in}}_{i,m,n,t} \leq T_t R^{\text{in}} \overline{R}_{i,n} : \theta^{\text{in}}_{i,m,n,t}, \forall i, m, n, t \tag{6b}$$

$$r^{\text{out}}_{i,m,n,t} \leq T_t R^{\text{out}} \overline{R}_{i,n} : \theta^{\text{out}}_{i,m,n,t}, \forall i, m, n, t \tag{6c}$$

$$\underline{\theta}_{i,m,n,t} : \underline{R}_{i,n} \overline{R}_{i,n} \leq r^{\text{sto}}_{i,m,n,t} \leq \overline{R}_{i,n} : \overline{\theta}_{i,m,n,t}, \forall i, m, n, t \tag{6d}$$

$$c_{m,n,t} - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{i,n}} y_{i,m,n,t,u} + V T_t \sum_{\ell \in \mathcal{L}^+_n} \hat{f}_{\ell,m,t} - V T_t \sum_{\ell \in \mathcal{L}^-_n} \hat{f}_{\ell,m,t} - \sum_{i \in \mathcal{I}} r^{\text{out}}_{i,m,n,t}$$
$$+ \sum_{i \in \mathcal{I}} r^{\text{in}}_{i,m,n,t} = 0 : \lambda_{m,n,t}, \forall m, n, t \tag{6e}$$

$$c_{m,n,t} \geq 0 : \phi^{c}_{m,n,t}, \forall m, n, t \tag{6f}$$

$$y_{i,m,n,t,u} \geq 0 : \phi^{y}_{i,m,n,t,u}, \forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{6g}$$

$$z_{i,n,u} \geq 0 : \phi^{z}_{i,n,u}, \forall i, n, u \in \mathcal{U}_{i,n} \tag{6h}$$

$$r^{\text{sto}}_{i,m,n,t} \geq 0 : \phi^{\text{sto}}_{i,m,n,t}, \forall i, m, n, t \tag{6i}$$

$$r^{\text{in}}_{i,m,n,t} \geq 0 : \phi^{\text{in}}_{i,m,n,t}, \forall i, m, n, t \tag{6j}$$

$$r^{\text{out}}_{i,m,n,t} \geq 0 : \phi^{\text{out}}_{i,m,n,t}, \forall i, m, n, t \tag{6k}$$

$$f_{j,\ell,m,t} \text{ u.r.s., } \forall \ell, m, t, j \in \mathcal{J}_\ell \tag{6l}$$

$$\hat{f}_{\ell,m,t} \text{ u.r.s., } \forall \ell, m, t \tag{6m}$$

$$v_{m,n,t} \text{ u.r.s., } \forall m, n, t \tag{6n}$$

Bi-Level Formulation: Lower Level's Strong Duality

$$\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} W_m \left[\sum_{n\in\mathcal{N}}\left(A_{m,n,t}c_{m,n,t} - \frac{1}{2}Z_{m,n,t}c^2_{m,n,t}\right) - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{opr}}_u y_{i,m,n,t,u}\right.$$

$$\left. - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}} C^{\text{sto}} r^{\text{out}}_{i,m,n,t}\right] - \sum_{i\in\mathcal{I}}\sum_{n\in\mathcal{N}}\sum_{u\in\mathcal{U}_{i,n}} C^{\text{gen}}_u z_{i,n,u}$$

$$- \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} W_m Z_{m,n,t}\sum_{i\in\mathcal{I}}\left(\sum_{u\in\mathcal{U}_{i,n}} y_{i,m,n,t,u} + r^{\text{out}}_{i,m,n,t} - r^{\text{in}}_{i,m,n,t}\right)^2$$

$$- HD\sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}} W_m F_u y_{i,m,n,t,u} \geq \frac{1}{2}\sum_{m\in\mathcal{M}} W_m \sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} Z_{m,n,t}c^2_{m,n,t}$$

$$+ \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}} T_t K^{\text{gen}}_{i,n,u}\left(\overline{G}_{m,n,t,u}\overline{\beta}_{i,m,n,t,u} - \underline{G}_{m,n,t,u}\underline{\beta}_{i,m,n,t,u}\right)$$

$$+ \sum_{\ell\in\mathcal{L}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}\sum_{j\in\mathcal{J}_\ell} T_t K^{\text{trn}}_{j,\ell}\left(\underline{\mu}_{j,\ell,m,t} + \overline{\mu}_{j,\ell,m,t}\right) + \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} \pi\left(\underline{\kappa}_{m,n,t} + \overline{\kappa}_{m,n,t}\right)$$

$$+ \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\sum_{u\in\mathcal{U}_{i,n}} T_t K^{\text{gen}}_{i,n,u}\left(R^{\text{up}}_u\beta^{\text{up}}_{i,m,n,t,u} + R^{\text{down}}_u\beta^{\text{down}}_{i,m,n,t,u}\right)$$

$$+ \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}} T_t\overline{R}_{i,n}\left(R^{\text{in}}\theta^{\text{in}}_{i,m,n,t} + R^{\text{out}}\theta^{\text{out}}_{i,m,n,t}\right)$$

$$+ \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{t\in\mathcal{T}}\left(\overline{R}_{i,n}\overline{\theta}_{i,m,n,t} - \underline{R}_{i,n}\overline{R}_{i,n}\underline{\theta}_{i,m,n,t}\right)$$

$$+ \sum_{i\in\mathcal{I}}\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\left(E^{\text{sto}}\right)^{T_1}\underline{R}_{i,n}\overline{R}_{i,n}\theta^{\text{sto}}_{i,m,n,1} \tag{7a}$$

Bi-Level Formulation: Lower Level's Dual Constraints

$$-W_m\left(A_{m,n,t}-Z_{m,n,t}c_{m,n,t}\right)+\lambda_{m,n,t}-\phi^{c}_{m,n,t}=0,\ \forall m,n,t \tag{8a}$$

$$W_m\left[C^{\text{opr}}_u+Z_{m,n,t}\left(\sum_{u'\in\mathcal{U}_{i,n}} y_{i,m,n,t,u'}+r^{\text{out}}_{i,m,n,t}-r^{\text{in}}_{i,m,n,t}\right)\right]+HDW_mF_u+\overline{\beta}_{i,m,n,t,u}$$
$$-\underline{\beta}_{i,m,n,t,u}+\beta^{\text{up}}_{i,m,n,t,u}-\beta^{\text{up}}_{i,m,n,t+1,u}+\beta^{\text{down}}_{i,m,n,t+1,u}-\beta^{\text{down}}_{i,m,n,t,u}-\lambda_{m,n,t}$$
$$-\phi^{y}_{i,m,n,t,u}=0,\forall i,m,n,t,u\in\mathcal{U}_{i,n} \tag{8b}$$

$$C^{\text{gen}}_u-\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}T_t\overline{G}_{m,n,t,u}\overline{\beta}_{i,m,n,t,u}+\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}T_t\underline{G}_{m,n,t,u}\underline{\beta}_{i,m,n,t,u}-\phi^{z}_{i,n,u}$$
$$-\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}T_tR^{\text{up}}_u\beta^{\text{up}}_{i,m,n,t,u}-\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}T_tR^{\text{down}}_u\beta^{\text{down}}_{i,m,n,t,u}=0,\forall i,n,u\in\mathcal{U}_{i,n} \tag{8c}$$

$$\mu_{j,\ell,m,t}+T_t\overline{\mu}_{j,\ell,m,t}-T_t\underline{\mu}_{j,\ell,m,t}-\psi_{\ell,m,t}=0,\ \forall\ell,m,t,j\in\mathcal{J}_\ell \tag{8d}$$

$$\psi_{\ell,m,t}+VT_t\lambda_{n^+_\ell,m,t}-VT_t\lambda_{n^-_\ell,m,t}=0,\ \forall\ell,m,t \tag{8e}$$

$$\sum_{j\in\mathcal{J}_\ell}\left(\sum_{\ell\in\mathcal{L}^-_n}x_{j,\ell}B_{j,\ell}\mu_{j,\ell,m,t}-\sum_{\ell\in\mathcal{L}^+_n}x_{j,\ell}B_{j,\ell}\mu_{j,\ell,m,t}\right)+\overline{\kappa}_{m,n,t}-\underline{\kappa}_{m,n,t}=0,\forall m,n,t \tag{8f}$$

$$\theta^{\text{sto}}_{i,m,n,t}-\left(E^{\text{sto}}\right)^{T_t}\theta^{\text{sto}}_{i,m,n,t+1}+\overline{\theta}_{i,m,n,t}-\underline{\theta}_{i,m,n,t}-\phi^{\text{sto}}_{i,m,n,t}=0,\ \forall i,m,n,t \tag{8g}$$

$$-W_mZ_{m,n,t}\left(\sum_{u\in\mathcal{U}_{i,n}}y_{i,m,n,t,u}+r^{\text{out}}_{i,m,n,t}-r^{\text{in}}_{i,m,n,t}\right)-E^{\text{in}}\theta^{\text{sto}}_{i,m,n,t}+\theta^{\text{in}}_{i,m,n,t}+\lambda_{m,n,t}$$
$$-\phi^{\text{in}}_{i,m,n,t}=0,\ \forall i,m,n,t \tag{8h}$$

Bi-Level Formulation: Lower Level's Dual Constraints (cont'd)

$$W_m \left[C^{\text{sto}} + Z_{m,n,t} \left(\sum_{u \in \mathcal{U}_{i,n}} y_{i,m,n,t,u} + r^{\text{out}}_{i,m,n,t} - r^{\text{in}}_{i,m,n,t} \right) \right] + \theta^{\text{sto}}_{i,m,n,t} + \theta^{\text{out}}_{i,m,n,t}$$

$$- \lambda_{m,n,t} - \phi^{\text{out}}_{i,m,n,t} = 0, \ \forall i, m, n, t \tag{9a}$$

$$\underline{\beta}_{i,m,n,t,u} \geq 0, \overline{\beta}_{i,m,n,t,u} \geq 0, \beta^{\text{up}}_{i,m,n,t,u} \geq 0, \beta^{\text{down}}_{i,m,n,t,u} \geq 0, \phi^{y}_{i,m,n,t,u} \geq 0,$$

$$\forall i, m, n, t, u \in \mathcal{U}_{i,n} \tag{9b}$$

$$\phi^{z}_{i,n,u} \geq 0, \ \forall i, n, u \in \mathcal{U}_{i,n} \tag{9c}$$

$$\underline{\mu}_{j,\ell,m,t} \geq 0, \overline{\mu}_{j,\ell,m,t} \geq 0, \ \forall \ell, m, t, j \in \mathcal{J}_{\ell} \tag{9d}$$

$$\underline{\kappa}_{m,n,t} \geq 0, \overline{\kappa}_{m,n,t} \geq 0, \phi^{c}_{m,n,t} \geq 0, \ \forall m, n, t \tag{9e}$$

$$\underline{\theta}_{i,m,n,t} \geq 0, \overline{\theta}_{i,m,n,t} \geq 0, \theta^{\text{in}}_{i,m,n,t} \geq 0, \theta^{\text{out}}_{i,m,n,t,u} \geq 0, \phi^{\text{sto}}_{i,m,n,t} \geq 0, \phi^{\text{in}}_{i,m,n,t} \geq 0,$$

$$\phi^{\text{out}}_{i,m,n,t} \geq 0, \ \forall i, m, n, t \tag{9f}$$

$$\lambda_{m,n,t} \text{ u.r.s., } \forall m, n, t \tag{9g}$$

$$\mu_{j,\ell,m,t} \text{ u.r.s., } \forall \ell, m, t, j \in \mathcal{J}_{\ell} \tag{9h}$$

$$\psi_{\ell,m,t} \text{ u.r.s., } \forall \ell, m, t \tag{9i}$$

$$\theta^{\text{sto}}_{i,m,n,t} \text{ u.r.s., } \forall i, m, n, t \tag{9j}$$

Linearisation of (8f)

Linearise (8f) via the auxiliary variable $\hat{\mu}_{j,\ell,s}$ as follows:

$$-\sum_{\ell\in\mathcal{L}_n^+}\sum_{j\in\mathcal{J}_\ell} B_{j,\ell}\left(\mu_{j,\ell,m,t}-\hat{\mu}_{j,\ell,m,t}\right)+\sum_{\ell\in\mathcal{L}_n^-}\sum_{j\in\mathcal{J}_\ell} B_{j,\ell}\left(\mu_{j,\ell,m,t}-\hat{\mu}_{j,\ell,m,t}\right)$$

$$+\overline{\kappa}_{m,n,t}-\underline{\kappa}_{m,n,t}=0,\forall m,n,t \tag{10a}$$

$$-x_{j,\ell}M^{\mathrm{trn}}\leq\mu_{j,\ell,m,t}-\hat{\mu}_{j,\ell,m,t}\leq x_{j,\ell}M^{\mathrm{trn}},\ \forall\ell,m,t,j\in\mathcal{J}_\ell \tag{10b}$$

$$-\left(1-x_{j,\ell}\right)M^{\mathrm{trn}}\leq\hat{\mu}_{j,\ell,m,t}\leq\left(1-x_{j,\ell}\right)M^{\mathrm{trn}},\ \forall\ell,m,t,j\in\mathcal{J}_\ell \tag{10c}$$

$$\hat{\mu}_{j,\ell,m,t}\ \text{u.r.s.},\ \forall\ell,m,t,j\in\mathcal{J}_\ell \tag{10d}$$